1. Solving Higher Degree Equations

Question:

Solve the following equations:

i) 3x - 2 = 0

ii) x^2 + x +1 = 0

iii) $2x^2 - 7x + 6 = 0$

iv) $x^3 - x^2 + 4x + 6 = 0$

v) $x^{12} - 6 = 0$

Aim:

To write a Mathematica program to solve higher degree equations.

Procedure:

- Click on **Mathematica** icon in the desktop.
- Open a new **Notebook** by clicking **New Document**.
- Enter the solving equation in Notebook.
- Then execute the equation by clicking **Shift + Enter** or by using the **Enter key** which is in Numeric key.
- Then we get the output in the Notebook.

```
In[1]:= Program :
         Solve[3 x - 2 == 0, x]
```

Out[1]= Program : $\{\{x \to \frac{2}{3}\}\}$

```
In[2]:= Solve[x^2 + x + 1 == 0, x]
```

Out[2]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

```
In[3]:= N[%]
```

Out[3]= $\{\{x \to -0.5 - 0.866025\, i\}, \{x \to -0.5 + 0.866025\, i\}\}$

```
In[4]:= Solve[2 x^2 - 7 x + 6 == 0, x]
```

Out[4]= $\{\{x \to \frac{3}{2}\}, \{x \to 2\}\}$

```
In[5]:= Solve[x^3 - x^2 + 4 x + 6  == 0, x]
```

Out[5]= $\{\{x \to -1\}, \{x \to 1 - i\sqrt{5}\}, \{x \to 1 + i\sqrt{5}\}\}$

```
In[6]:= Solve[x^12 - 6 == 0, x]
```

Out[6]= $\{\{x \to -6^{1/12}\}, \{x \to -i\, 6^{1/12}\}, \{x \to i\, 6^{1/12}\}, \{x \to 6^{1/12}\},$
$\{x \to -(-1)^{1/6} 6^{1/12}\}, \{x \to (-1)^{1/6} 6^{1/12}\}, \{x \to -(-1)^{1/3} 6^{1/12}\}, \{x \to (-1)^{1/3} 6^{1/12}\},$
$\{x \to -(-1)^{2/3} 6^{1/12}\}, \{x \to (-1)^{2/3} 6^{1/12}\}, \{x \to -(-1)^{5/6} 6^{1/12}\}, \{x \to (-1)^{5/6} 6^{1/12}\}\}$

Conclusion:

Thus, the Mathematica program for solving higher degree equations was implemented successfully.

2. Solving System of Equations by Matrix Method and and Finding the Eigen Values and Eigen Vector of a Matrix of Order 4 x 4

Question:

Solve the following system of equation:

$-2w + y + z = -3$

$x + 2y - z = 2$

$-3w + 2x + 4y + z = -2$

$-w + x - 4y - 7z = -19$

Aim:

To write a Mathematica program to solve system of equation by matrix method and also find the eigen values and eigen vectors of the given system.

Procedure:

- Click the Mathematica icon in the desktop.
- Open a new Notebook by clicking New Document
- Give the values of the matrices A and B.
- Call the function LinearSolve, Eigenvalues and Eigenvectors for solving given system of equations, getting eigen values and eigen vectors of A respectively.
- Then we get the output in the Notebook.

Program:

```
In[]:= A = {{-2, 0, 1, 1}, {0, 1, 2, -1}, {-3, 2, 4, 1}, {-1, 1, -4, -7}};
B = {{-3}, {2}, {-2}, {-19}};
StringForm["A = ``.", MatrixForm[A]]
StringForm["B = ``.", MatrixForm[B]]
```

Out[]= $A = \begin{pmatrix} -2 & 0 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ -3 & 2 & 4 & 1 \\ -1 & 1 & -4 & -7 \end{pmatrix}.$

Out[]= $B = \begin{pmatrix} -3 \\ 2 \\ -2 \\ -19 \end{pmatrix}.$

```
In[]:= {{w}, {x}, {y}, {z}} = LinearSolve[A, B];
StringForm["w = ``, x = ``, y = ``, and z = ``.", w, x, y, z]
```

Out[]= w = 3, x = -1, y = 2, and z = 1.

In[ ]:=
```
e = Eigenvalues[A];
StringForm["The eigen values are \n e1 =``, \n e2 =``, \n e3 =``,\n e4 =``.",
 e[[1]], e[[2]], e[[3]], e[[4]]]
v = Eigenvectors[A];
StringForm["The eigen vectors are \n v1 =``, \n v2 =``, \n v3 =``,\n v4 =``.",
 v[[1]], v[[2]], v[[3]], v[[4]]]
```

Out[ ]= The eigen values are

e1 $= -4 - \sqrt{3}$,

e2 $= 2 + \sqrt{7}$,

e3 $= -4 + \sqrt{3}$,

e4 $= 2 - \sqrt{7}$.

Out[ ]= The eigen vectors are

v1 $= \left\{-\frac{8-\sqrt{3}}{21+5\sqrt{3}}, -\frac{-8+\sqrt{3}}{21+5\sqrt{3}}, -\frac{8-\sqrt{3}}{21+5\sqrt{3}}, 1\right\}$,

v2 $= \left\{-\frac{3\left(2+\sqrt{7}\right)}{5+13\sqrt{7}}, -\frac{5\left(14+\sqrt{7}\right)}{5+13\sqrt{7}}, -\frac{50+31\sqrt{7}}{5+13\sqrt{7}}, 1\right\}$,

v3 $= \left\{-\frac{-8-\sqrt{3}}{-21+5\sqrt{3}}, -\frac{8+\sqrt{3}}{-21+5\sqrt{3}}, -\frac{-8-\sqrt{3}}{-21+5\sqrt{3}}, 1\right\}$,

v4 $= \left\{-\frac{3\left(-2+\sqrt{7}\right)}{-5+13\sqrt{7}}, -\frac{5\left(-14+\sqrt{7}\right)}{-5+13\sqrt{7}}, -\frac{-50+31\sqrt{7}}{-5+13\sqrt{7}}, 1\right\}$.

Conclusion:

Thus, the Mathematica program for solving system of equation by matrix method and also for finding the eigen values and eigen vectors was implemented successfully.

3. Solving System of Non-Linear Equations

Question:

Solve the following system of equations:

x y - 5y + 10 = 0

$x^3 - y^2 = 2$

with initial condition x = 1, y = 1.

Aim:

To write a Mathematica program to solve system of non-linear equations.

Procedure:

- Click the **Mathematica** icon in the desktop.
- Open a new **Notebook** by clicking **New Document.**
- Use the function **FindRoot** for solving given non-linear equation with the given initial value of x and y.
- Then we get the output in the Notebook.

Program:

```
In[7]:= f[u_, v_] = {u v - 5 v + 10, u^3 - v^2 - 2};
        f[x, y]
        FindRoot[f[x, y], {{x, 1}, {y, 1}}]
Out[8]= {10 - 5 y + x y, -2 + x^3 - y^2}
Out[9]= {x → 3., y → 5.}

In[10]:= f[3, 5]
Out[10]= {0, 0}
```

Conclusion:

Thus, the Mathematica program for solving system of non-linear equation was implemented successfully.

4. Finding Second and Third Order Derivative of Different Functions

Question:

Find the second and third order derivatives of given function with respect to their variables:

(i) f = t sin(5x)

(ii) $g = e^t x^2$

Aim:

To write the Mathematica program for finding second and third order derivatives of the given functions.

Procedure:

- Click on **Mathematica** icon in desktop.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- Use **D** function to find the second and third order derivatives of the given function.
- After executing the **D** function, we get the output in the Notebook.

Program:

```
In[11]:= f = t Sin[5 x]
         d1fx = D[f, x]
Out[11]= t Sin[5 x]

Out[12]= 5 t Cos[5 x]

In[13]:= d2fx = D[f, {x, 2}]
Out[13]= -25 t Sin[5 x]

In[14]:= d3fx = D[f, {x, 3}]
Out[14]= -125 t Cos[5 x]

In[15]:=

In[16]:= d1ft = D[f, t]
Out[16]= Sin[5 x]

In[17]:= d2ft = D[f, {t, 2}]
Out[17]= 0

In[18]:= g = Exp[t] x^2
         d2gx = D[g, {x, 2}]
Out[18]= e^t x^2

Out[19]= 2 e^t
```

In[20]:=
```
d3gt = D[g, {t, 3}]
```
Out[20]= $e^t x^2$

Conclusion:

Thus, the Mathematica program for finding second and third order derivatives of the given different functions was implemented successfully.

5. Finding the Integration of Different Functions with Limits

Question:

Find the integration of given functions:

i) f = x^7 with lower limit a = 0 and upper limit b =1.

ii) g = 1/x with lower limit a = 1 and upper limit b =2.

iii) h = $\sqrt{x}$ Log[x] with lower limit a = 0 and upper limit b =1.

iv) z = e^{-x^2} with lower limit a = 0 and upper limit b = ∞.

Aim:

To write the Mathematica program for finding integration of the given functions.

Procedure:

- Click on **Mathematica** icon in desktop.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- Use **Integrate** function to find integration of the given function.
- After executing the **Integrate** function, we get the output in the Notebook.

Program:

```
In[21]:= f = x^7
         F = Integrate[f, {x, 0, 1}]
```

Out[21]= x^7

Out[22]= $\frac{1}{8}$

```
In[23]:= g = 1/x
         G = Integrate[g, {x, 1, 2}]
```

Out[23]= $\frac{1}{x}$

Out[24]= Log[2]

```
In[25]:= h = Sqrt[x] Log[x]
         H = Integrate[h, {x, 0, 1}]
```

Out[25]= $\sqrt{x}$ Log[x]

Out[26]= $-\frac{4}{9}$

```
In[27]:= z = Exp[-x^2]
         Z = Integrate[z, {x, 0, Infinity}]
```

Out[27]= e^{-x^2}

Out[28]= $\frac{\sqrt{\pi}}{2}$

Conclusion:

Thus, the Mathematica program for finding the integration of the given different functions was implemented successfully.

6. Evaluation of Double and Triple Integrals

Find the integration of given functions:

(i) Integrate the following function f(x,y) = $\frac{1}{\sqrt{x+y}\,(1+x+y)^2}$ over the triangular region bounded by $0 \leq x \leq$

,

and $0 \leq y \leq 1 - x$.

(ii) Integrate the function over the region f(x,y,z) = y sinx + z cosx over the region $0 \leq x \leq \pi$, $0 \leq y \leq 1$, and $-1 \leq z \leq 1$

Aim:

To write the Mathematica program for evaluating double and triple integrals.

Procedure:

- Click on **Mathematica** icon in desktop.
- Select the command window.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- Use **Integrate** function with given limits to evaluate double and triple integral respectively.
- After executing the **Integrate** function, we get the output in the Notebook.

Program

```
In[53]:= f = 1/(Sqrt[x + y] (1 + x + y)^2)
         ymax = 1 - x;
         ans = Integrate[f, {x, 0, 1}, {y, 0, ymax}]
```

Out[53]= $\frac{1}{\sqrt{x+y}\,(1+x+y)^2}$

Out[55]= $\frac{1}{4}(-2+\pi)$

```
In[56]:= N[%, 4]
Out[56]= 0.2854
```

```
In[57]:= Clear[x, y, g, f, z]
         g = y Sin[x] + z Cos[x]
         ans2 = Integrate[g, {x, 0, Pi}, {y, 0, 1}, {z, -1, 1}]
Out[58]= z Cos[x] + y Sin[x]
Out[59]= 2
```

Conclusion:

Thus, the Mathematica program for evaluating the double and triple integrals was implemented successfully.

7. Solving Ordinary Differential Equations with Initial Conditions

Question:

Solve the equation $\frac{dy}{dt} = ty$ with initial condition y(0)=2 .

Aim:

To write the Mathematica program for solving ODE with initial conditions.

Procedure:

- Click on **Mathematica** icon in desktop.
- Select the command window.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- Use **DSolve** function with given initial condition for y with respect to t.
- After executing the **DSolve** function, we get the output in the Notebook.

Program:

```
In[60]:= ClearAll[Derivative]
Clear[x, y, t]
ode = t y[t]
ans = DSolve[{y'[t] == ode, y[0] == 2}, y[t], t]
Out[62]= t y[t]
```

Out[63]= $\left\{\left\{y[t] \to 2\, e^{\frac{t^2}{2}}\right\}\right\}$

Conclusion:

Thus, the Mathematica program for solving the ordinary differential equation was implemented successfully.

8. Solving System of Ordinary Differential Equations

Question:

Solve the following system of ordinary differential equation

$$\frac{du}{dt} = 3\,u + 4\,v \text{ and } \frac{dv}{dt} = -4\,u + 3\,v$$

with initial conditions $u(0) = 0$ and $v(0) = 1$.

Aim:

To write the Mathematica program for solving system of ODE with initial conditions.

Procedure:

- Click on **Mathematica** icon in desktop.
- Select the command window.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- Use **DSolve** function with given initial condition for u and v with respect to t.
- After executing the **DSolve** function, we get the output in the Notebook.

Program:

```
In[64]:= ClearAll[Derivative]
         Clear[u, v, x, ans]
         odeu = 3 u[t] + 4 v[t]
         odev = -4 u[t] + 3 v[t]
         ans = DSolve[{u'[t] == odeu, v'[t] == odev, u[0] == 0, v[0] == 1}, {u[t], v[t]}, t]
Out[66]= 3 u[t] + 4 v[t]
Out[67]= -4 u[t] + 3 v[t]
Out[68]= {{u[t] → e^(3 t) Sin[4 t], v[t] → e^(3 t) Cos[4 t]}}
```

Out[68]= $\{\{u[t] \to e^{3t}\,\text{Sin}[4\,t],\ v[t] \to e^{3t}\,\text{Cos}[4\,t]\}\}$

Conclusion:

Thus, the Mathematica program for solving the system of ordinary differential equation was implemented without any error and the output is displayed in the notebook successfully.

9. Creating and Plotting 2-D and 3-D Graphs

Question:

i. Create 2-D graph of y1 = sin(x) and y2 = cos(x) where $0 \le x \le 30\pi$

ii. Create 3-D graph of x = (3 + Cos[$\sqrt{32}$ *t*])Cos[*t*], y = Sin[$\sqrt{32}$ *t*] and z = (3 + Cos[$\sqrt{32}$ *t*]) Sin[*t*] where $0 \le t \le 30\pi$

Aim:

To write the Mathematica program for creating and plotting 2-D and 3-D graphs.

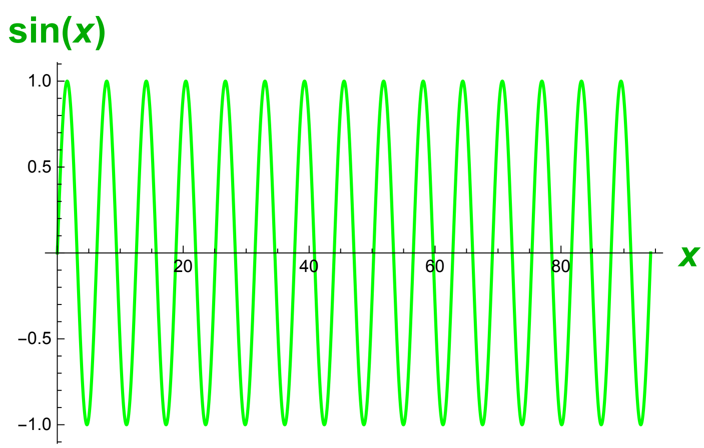
Procedure:

- Click on **Mathematica** icon in desktop.
- Select the command window.
- Open a new **Notebook** by clicking **New Document**.
- Enter the given functions in the Notebook.
- use **Plot** and **ParametricPlot3D** functions for plotting 2-D and 3-D graph respectively
- After executing the **Plot** and **ParametricPlot3D** function, we get the output in the Notebook.

Program:

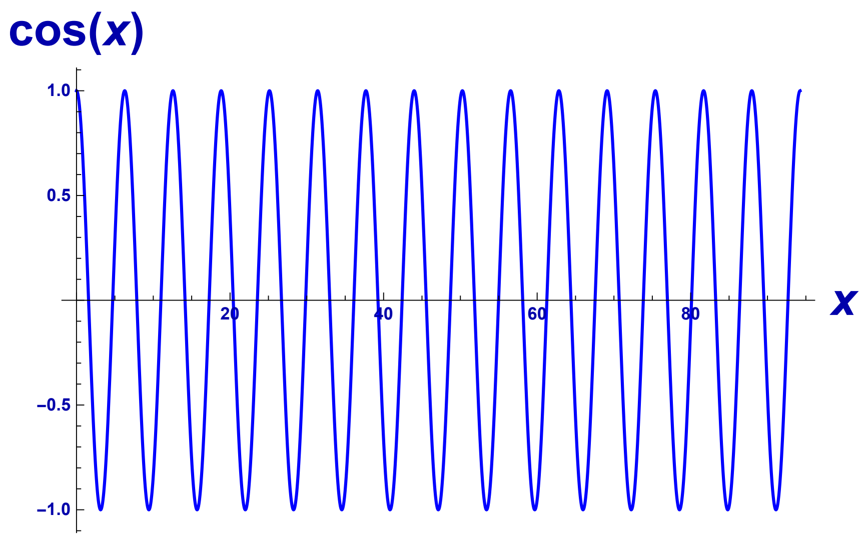
In[69]:=
```
Clear[y1, y2, x]
y1 = Sin[x];
y2 = Cos[x];
Plot[y1, {x, 0, 30 Pi}, AxesLabel → {Style[x, Bold, FontSize → 19, Darker[Green]],
   Style[y1, Bold, FontSize → 19, Darker[Green]]}, PlotStyle → Directive[Green]]
```

Out[72]=



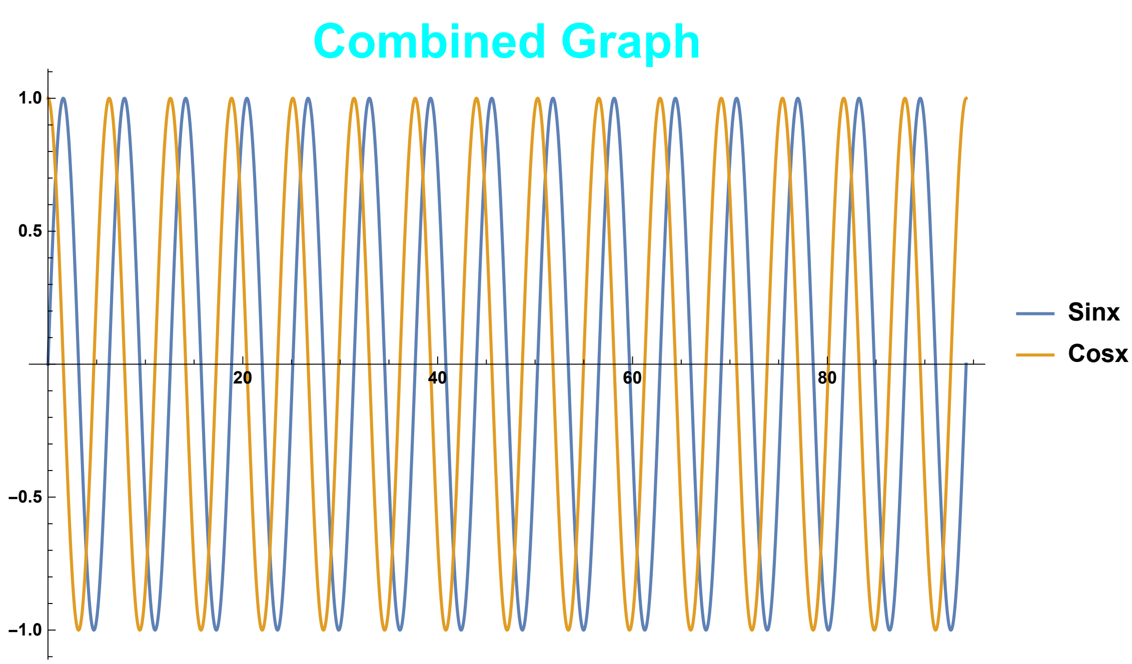
In[73]:=
```
Plot[y2, {x, 0, 30 Pi},
 AxesLabel → {Style[x, Large, Darker[Blue]], Style[y2, Large, Darker[Blue]]},
 LabelStyle → Directive[Bold, Darker[Blue]], PlotStyle → Directive[Blue]]
```

Out[73]=



In[74]:=
```
Plot[{y1, y2}, {x, 0, 30 Pi}, PlotLegends → {Sinx, Cosx},
 PlotLabel → Style["Combined Graph", FontSize → 25, FontColor → Hue[.5]],
 LabelStyle → Directive[Bold]]
```

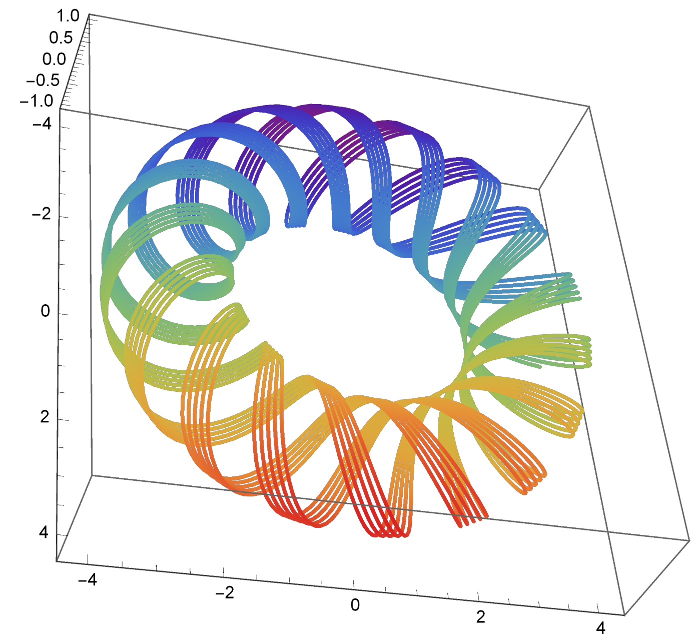
Out[74]=



Plotting 3D Graph

In[75]:=
```
Clear[x, y, z, t]
x = (3 + Cos[Sqrt[32] t]) Cos[t];
y = Sin[Sqrt[32] t];
z = (3 + Cos[Sqrt[32] t]) Sin[t];
ParametricPlot3D[{x, y, z}, {t, 0, 30 Pi}, ColorFunction → "Rainbow"]
```

Out[79]=



Conclusion:

Thus, the Mathematica program for creating and plotting 2-D and 3-D graph was implemented without any error and the output is displayed in the notebook successfully.

10. Solving Linear Programming Problems

Question:

Maximize $z = 4x_1 + 3x_2$

Subject to the constraints,

$2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$0 \leq x_1 \leq 400$

$0 \leq x_2 \leq 700$

Aim:

To write the Mathematica program for creating and plotting 2-D and 3-D graphs.

Procedure:

- Click on **Mathematica** icon in desktop.
- Select the command window.
- Open a new **Notebook** by clicking **New Document**.
- use **Maximize** function for solving given LPP problem
- After executing the **Maximize** function, we get the output in the Notebook.

Program:

```
In[80]:= Clear[x, y]
         {max, sol} = Maximize[{4 x + 3 y, 2 x + y ≤ 1000, x + y ≤ 800, 0 ≤ x ≤ 400, 0 ≤ y ≤ 700}, {x, y}]
Out[81]= {2600, {x → 200, y → 600}}

In[82]:= {2600, {x → 200, y → 600}}
         StringForm["The optimatl solution is = `` with x = `` and y = ``.",
          max, sol[[1]], sol[[2]]]
Out[82]= {2600, {x → 200, y → 600}}

Out[83]= The optimatl solution is = 2600 with x = x → 200 and y = y → 600.
```

Conclusion:

Thus, the Mathematica program for solving linear programming problem was implemented without any error and the output is displayed in the notebook successfully.